Unit 6 Day 3 Inverses Classwork Name: ______________________________

1. In regular arithmetic, 27 + 0 = 27. We call 0 the "additive identity."

   What matrix would act like the additive identity?

Verify that $\begin{bmatrix} -1 & 4 \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 6 & -3 \end{bmatrix}$ Verify that $\begin{bmatrix} -1 & 4 & 5 \\ -9 & 0 & 3 \\ 1 & 2 & 8 \end{bmatrix} + \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ -9 & 0 & 3 \\ 1 & 2 & 8 \end{bmatrix}$

This matrix is called the "zero matrix."

2. In regular arithmetic, 27(1) = 27. We call 1 the "multiplicative identity."

   Find a matrix that would act like the multiplicative identity: (This matrix is called the "identity matrix.")

Verify that $\begin{bmatrix} -1 & 4 \\ 6 & -3 \end{bmatrix} \cdot \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 6 & -3 \end{bmatrix}$ Verify that $\begin{bmatrix} -1 & 4 & 5 \\ -9 & 0 & 3 \\ 1 & 2 & 8 \end{bmatrix} \cdot \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ -9 & 0 & 3 \\ 1 & 2 & 8 \end{bmatrix}$

3. Is there a matrix such that $\begin{bmatrix} -1 & 4 \\ -9 & 0 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ -9 & 0 \\ 1 & 2 \end{bmatrix}$? What needs to be true about every identity matrix?

The 2 x 2 identity matrix is denoted by I_2; the 3 x 3 identity matrix is I_3, etc.

4. In regular arithmetic, when two numbers added together equal 0 (the additive identity), they are called <u>additive inverses</u>. Find the additive inverse of $\begin{bmatrix} -1 & 4 & 5 \\ -9 & 0 & 3 \\ 1 & 2 & 8 \end{bmatrix}$. Verify that $\begin{bmatrix} -1 & 4 & 5 \\ -9 & 0 & 3 \\ 1 & 2 & 8 \end{bmatrix} + \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. In regular arithmetic, when two numbers multiplied together equal 1 (the multiplicative identity), they are called reciprocals, or <u>multiplicative inverses</u>. Note that an exponent of -1 denotes a multiplicative inverse; $x \cdot x^{-1} = 1$

Remember that $5 \cdot 5^{-1} = 5 \cdot \frac{1}{5} = 1$. Verify that $\frac{3}{4} \cdot \left(\frac{3}{4}\right)^{-1} = 1$

Finding a multiplicative inverse of a matrix is more involved than the process was in regular arithmetic.
Only square matrices have inverses, and if the determinant equals 0, then no inverse exists.
The inverse of matrix M is denoted by M^{-1}.

How to find the inverse of a 2 x 2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$:

1. Find the determinant, which is $ad - bc$. Remember that value; you'll use it later.
2. In your original matrix, exchange elements a and d, then change the signs of elements b and c.
   Now your matrix looks like $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.
3. Find the reciprocal of the determinant and multiply the new matrix by that number.
   Thus <u>**a formula for finding the inverse of a 2 x 2 matrix**</u> is $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

6. Find the inverses of these matrices using the formula on the first page:

a. $\begin{bmatrix} 8 & -3 \\ -5 & 2 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 5 \\ 0 & -3 \end{bmatrix}$ c. $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

7. If $B = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$, find B^{-1}. Then show that $B \cdot B^{-1} = B^{-1} \cdot B = I_2$

8. The formula you used above is derived from a longer process. When the identity matrix is augmented on the right to an invertible matrix (the determinant is nonzero, so an inverse to the matrix does exist), and row-reduction is performed until the identity matrix appears on the left of the augmented matrix, then the remaining 2 x 2 matrix on the right is the inverse of the original.

Use the method described to find the inverse of $M = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$.

Step one is to augment I_2 to M: $\left[\begin{array}{cc|cc} 4 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right]$.

Now perform row operations until the augmented matrix looks like $\begin{bmatrix} 1 & 0 & & \\ 0 & 1 & & \end{bmatrix}$. Then prove you found the inverse by multiplying $M \cdot M^{-1} = M^{-1} \cdot M = I_2$.

9. In regular algebra, to solve a simple one-step equation such as 5x = 10, you would simply divide both sides by 5. But what if there was no such thing as division? You could solve this equation by multiplying each side by $\frac{1}{5}$, which is the reciprocal (or multiplicative inverse) of 5.

There is no division in matrix algebra. But you can multiply by an inverse to solve equations involving matrices.

This system of equations $\begin{cases} 5x + 3y = 9 \\ 8x + 5y = 14 \end{cases}$ can be represented by this matrix equation: $\begin{bmatrix} 5 & 3 \\ 8 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 14 \end{bmatrix}$

We can think of a simpler version of this matrix equation as AX = C where A is the square matrix of coefficients, X is the one column matrix of the variables, and C is the one column matrix of the constants.

Just as you solved the equation 5x = 10 by multiplying each side of the equation by the reciprocal of 5, you can solve the matrix equation AX = C by using the inverse of A to “undo” the multiplication of A.

When multiplying each side of the equation by A^{-1}, it is important to remember that matrix multiplication is not commutative, so A^{-1} needs to be on the left of each expression.

To solve a matrix equation of the form AX = C, multiply each side by A^{-1} so it would look like $A^{-1}AX = A^{-1}C$, which would simplify to $X = A^{-1}C$.

To solve this system of equations $\begin{cases} 5x + 3y = 9 \\ 8x + 5y = 14 \end{cases}$ by using inverse matrices,

begin by writing the matrix equation: $\begin{bmatrix} 5 & 3 \\ 8 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 14 \end{bmatrix}$.

Then find the inverse of matrix A = $\begin{bmatrix} 5 & 3 \\ 8 & 5 \end{bmatrix}$ by using the formula on the front page of the classwork.

A^{-1} =

Now multiply each side of the equation by A^{-1}, keeping it on the left side of each expression:

$$\begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 8 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} 9 \\ 14 \end{bmatrix}$$

You should end up with a matrix equation like this: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$. What does it mean?

Practice solving a few more systems using inverse matrices. (You already found the inverses earlier on this classwork.)

10. $\begin{cases} 4x + 2y = 30 \\ 5x + 3y = -11 \end{cases}$

11. $\begin{cases} x + 5y = 78 \\ -3y = 84 \end{cases}$

12. $\begin{cases} 8x - 3y = 0.6 \\ -5x + 2y = 1.8 \end{cases}$