Unit 3 Day 4 – Linear vs Exponential Classwork Name: ______________________________________

One day in the ancient kingdom of Montarek, a peasant saved the life of the king's daughter. The king was so grateful he told the peasant she could have any reward she desired. The peasant made an unusual request:

"I would like you to place 1 ruba on the first square of my chessboard, 2 rubas on the second square, 4 on the third square, 8 on the fourth square, and so on. Continue this pattern until you have covered all 64 squares. Each square should have twice as many rubas as the previous square."

The king replied, "Rubas are the least valuable coin in the kingdom. Surely you can think of a better reward!" But the peasant insisted, so the king agreed to her request. Did the peasant make a wise choice? (Fill out Plan 1 below.)

The patterns of change in the ballot problem and the king's Rubas pattern are geometric sequences, which show exponential growth. The "pattern" or the number you multiplied by each time was called the ratio in geometric sequences, but in exponential relationships this fixed number is called the ***growth factor***.

The king told the queen about the reward he had promised the peasant. The queen said, "You have promised her more money than we have in the entire royal treasury! You must convince her to accept a different reward."

After much thought, the king came up with Plan 2. He would make a new board with only 16 squares. He would place 1 Ruba on the first square and 3 Rubas on the second. He drew a graph to show the number of Rubas on the first five squares. He would continue this pattern until all 16 squares were filled.

The queen wasn't convinced about the king's new plan, so she devised a third plan. Under Plan 3, the king would make a board with 12 squares. He would place 1 Ruba on the first square. He would use the equation $R = 4^{n-1}$ to figure out how many Rubas to put on each of the other squares. In the equation, R is the number of Rubas on square n.

Before presenting Plans 2 and 3 to the peasant, the king consulted with his financial advisors. They told him that either plan would devastate the royal treasury. The advisors proposed a fourth plan. Under Plan 4, the king would put 20 Rubas on the first square of a chessboard, 25 on the second, 30 on the third, and so on. He would increase the number of Rubas by 5 for each square, until all 64 squares were covered.

In this table, Plan 1 is the reward requested by the peasant. Plan 2 is the king's new plan. Plan 3 is the queen's plan. Plan 4 is the advisors' plan. Complete the table to show the number of Rubas on squares 1 to 10 for each plan. Also show the amount for the final square of each plan, to see who has the best plan.

Square	Plan 1 Rubas	Plan 2 Rubas	Plan 3 Rubas	Plan 4 Rubas
1	1	1	1	
2	2	3	4	
3	4	9	16	
4	8	27	64	
5	16	81		
6	32			
7				
8				
9				
10				
Final Square:				

1. Explain the differences between the plans; how are the patterns of change different and similar?

2. Are the growth patterns for Plans 1, 2 and 3 exponential relationships?
If so, what is the growth factor for each? Is Plan 4 exponential? Explain:

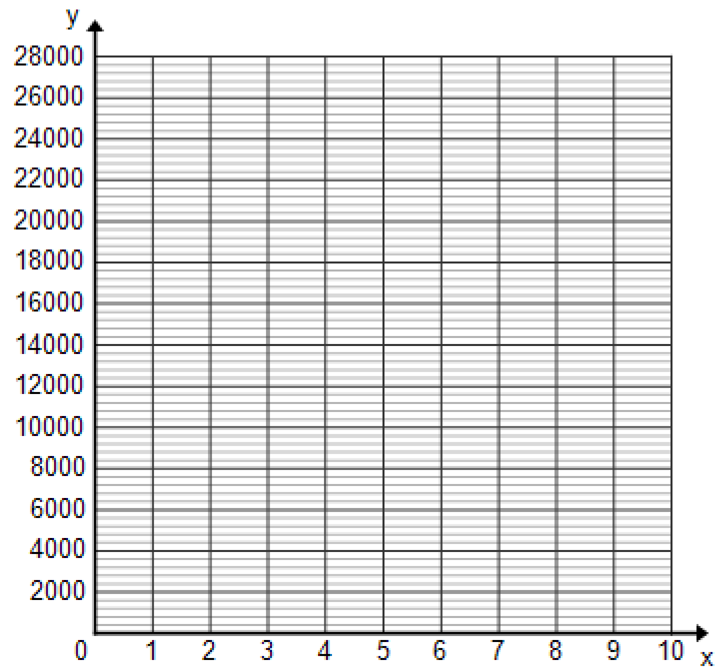
3. Write an explicit equation for the relationship between the number of the square n and the number of Rubas R for each plan.

Plan 1:

Plan 2:

Plan 3:

Plan 4:



4. Make a graph of Plan 1 for n = 1 to 10. Do not connect the dots. (why?)

5. Use a different color to graph Plan 2 on the same axes.

Use a third color to graph Plan 3.

Use a fourth color to graph Plan 4.

6. Write a sentence comparing and contrasting the graphs.

7. Which square will have 2^{30} rubas for Plan 1? Explain.

8. What is the first square on which the king will place at least one million rubas for Plan 1? How many rubas will be on this square?

9. The queen's assistant wrote the equation $R = \frac{1}{4}(4^n)$ for Plan 3. This equation is different than the one the queen wrote. Did the assistant make a mistake? Explain.

10. How many Rubas would be on square 20 for Plan 1?

How many Rubas would be on square 20 for Plan 4?