Name: _____ Date: _____ Period: _____

Sec 1H Unit 7 Day 5 - Determinants Classwork

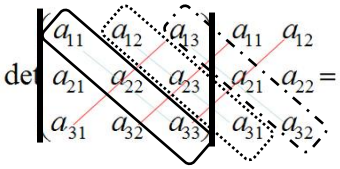
Square matrices have a useful value that can be computed, called the **determinant**. The notation for a determinant is to use straight lines instead of brackets.

The determinant of a 2 x 2 matrix is defined as $\begin{vmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{vmatrix} = \boldsymbol{ad - bc}$.

1. Find the determinant:

a. $\begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix}$ b. $\begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}$ c. $\begin{vmatrix} -1 & 2 \\ -4 & 8 \end{vmatrix}$

2. The determinant of a 3 x 3 matrix is slightly more complicated:



$$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - (a_{31}a_{22}a_{13} + a_{32}a_{23}a_{11} + a_{33}a_{21}a_{12})$$

Matrices larger than 3 x 3 need a different algorithm to find the determinant, and we won't be studying it in this class. (You may use technology if necessary for any larger matrices.)

Find the determinant:

a. $\begin{vmatrix} -8 & -7 & 3 \\ 6 & 2 & -3 \\ 1 & -5 & -2 \end{vmatrix}$ b. $\begin{vmatrix} 5 & -2 & -7 \\ -1 & 8 & 3 \\ 0 & 2 & 4 \end{vmatrix}$ c. $\begin{vmatrix} 4 & 6 & 1 \\ 10 & 0 & -2 \\ -1 & -4 & 7 \end{vmatrix}$

Let's discover what the determinant can tell us. Recall this problem from your homework on Unit 4, Day 4:

Kristin and Trevon each improved their yards by planning rose bushes and shrubs. They bought their supplies from the same store. Kristin spent $42 on 3 rose bushes and 3 shrubs. Trevon spent $54 on 7 rose bushes and 3 shrubs.

3. Write a system of equations to represent the situation, then write an augmented matrix based on the system of equations:

4. When we solved this problem using substitution or elimination, we learned that each rose was $3 and the cost of each shrub was $11. Solve the system by row-reduction to verify the solution of (3, 11).

5. What does it mean on a graph when a system of equations has a solution? When it doesn't have a solution? What does it mean on a graph when a system has infinitely many solutions?

6. Imagine that Kristin told her friend Katy about the roses and shrubs, then Katy went to the same store and bought 6 roses and 6 shrubs. She spent $84. How does this compare to what Kristin purchased and how much she spent?

7. Write a system of equations representing both Kristin and Katy's purchases. Solve the system.

8. Find the determinant of this matrix $\begin{bmatrix} 3 & 3 \\ 7 & 3 \end{bmatrix}$ based on Kristin's and Trevon's system of equations: $\begin{vmatrix} 3 & 3 \\ 7 & 3 \end{vmatrix} =$

9. Now find the determinant of this matrix $\begin{bmatrix} 3 & 3 \\ 6 & 6 \end{bmatrix}$ based on Kristin's and Katy's system of equations: $\begin{vmatrix} 3 & 3 \\ 6 & 6 \end{vmatrix} =$

10. Write a conjecture about what the value of the determinant might tell us:

11. Imagine that Boris lives across the street from Kristin, and he wants to make his yard look better than hers. He went to a different store and purchased 6 roses and 6 shrubs, but he paid $90. How does this compare to what Kristin bought and how much she spent?

12. Write a system of equations representing both Kristin's and Boris's purchases. Solve the system.

13. Find the determinant of this matrix $\begin{bmatrix} 3 & 3 \\ 6 & 6 \end{bmatrix}$ based on Kristin's and Boris's system of equations: $\begin{vmatrix} 3 & 3 \\ 6 & 6 \end{vmatrix} =$

14. What do you notice about the matrix from #9 and the one from #13?

15. If necessary, revise your conjecture about what a determinant can tell us: