Unit 1 Day 6 - Solving Systems using Row Reduction Classwork Name ________________________________

NOTES: The principles involved in row reduction of matrices are equivalent to those we used in the elimination method of solving systems of equations. That is, we are allowed to

- Multiply a row by a non-zero constant.
- Add one row to another.
- Interchange between rows
- Add a multiple of one row to another.
- Keep a row unchanged

How do we use this to **solve systems of equations?** We follow the steps:

Step 1 Write each equation in standard form ($ax + by = c$)
Step 2 Write the augmented matrix of the system.
Step 3 Row reduce the augmented matrix.
Step 4 Write the new, equivalent, system that is defined by the new, row reduced, matrix.
Step 5 Solution is found by going from the bottom equation

An **augmented matrix** consists of the coefficients and constant terms of a system of linear equations.

$$\begin{cases} 7x + 3y = 4 \\ 2x - 3y = 10 \end{cases} \qquad \left[\begin{array}{cc|c} 7 & 3 & 4 \\ 2 & -3 & 10 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -2 \end{array}\right]$$

A vertical line separates the coefficients from the constants.

Goal

Row reduction is the process of performing elementary row operations on an augmented matrix to solve a system. The goal is to get the coefficients to reduce to the identity matrix on the left side. This is called **reduced row-echelon form**.

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -2 \end{array}\right] \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \begin{array}{c} 1x + 0y = 2 \\ 0x + 1y = -2 \end{array} \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \begin{array}{c} x = 2 \\ y = -2 \end{array}$$

<u>NOW TRY IT</u>: Find a sequence of matrices that starts with the original matrix and ends with the solution matrix. Justify each step with notation.

$$\begin{bmatrix} 5 & 2 & 43.50 \\ 2 & 4 & 35 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 6.5 \\ 0 & 1 & 5.5 \end{bmatrix}$$

Solve by using row-reduction.

1. $\begin{cases} -2x = y \\ 2 - y = x \end{cases}$

2. $\begin{cases} 2x + y = 11 \\ 3x - 2y = 6 \end{cases}$

3. $\begin{cases} 4x + 4y = 32 \\ x + 3y = 16 \end{cases}$